Ill Conditioning in Self-Heating FET Models

S. A. Maas, *Fellow, IEEE*

***Abstract*—This paper shows that the introduction of self heating effects into a FET model can cause severe ill conditioning in nonlinear circuit simulators. Depending on the parameter values of the model, the solutions can be indistinct, multiple, or even nonexistent. The result is poor convergence in nonlinear simulations using such models.**

I. Introduction

ALTHOUGH there has been considerable work on thermal stability in solid-state devices [1]–[10], there has been considerably less that examines the interaction between self-heating models and nonlinear circuit simulation. In harmonic-balance simulations of FET circuits that included self-heating effects, the author occasionally noticed poor convergence in models that were generally robust. Among these was the Motorola Electrothermal Model (MET) [10], [11]. The nonconvergence was clearly related to the self-heating; removing the self-heating effect, either by making the thermal resistance zero or by setting the temperature coefficients to zero, resulted in robust and rapid convergence. The sign of the temperature coefficient for the threshold voltage had an especially strong influence on this effect.

In closer investigation, we discovered that it is rather easy to create a situation where the MET model, or any other FET model, is ill-conditioned. In this case, a solution of the device equations may be poorly defined, and there also may be multiple solutions. Under some circumstances, no solutions are possible. If the model's thermal corrections are valid over a wide temperature range, such a situation implies that the device is thermally unstable; often, it is prone to thermal runaway. This situation exists for all methods of simulation.

II. Theory

Self heating in solid state devices is traditionally modeled by generating a current that is numerically equal to the power in the device, and applying that current to a parallel RC circuit. The resistance of the RC circuit models the thermal conductivity, and the capacitance, the thermal mass. The voltage across this combination is the temperature increase in the device; thus, for dc bias, the temperature increase $\Delta \mathrm{T}$ is

$$\Delta T = P_d R_{\mathrm{th}} \tag{1}$$

where P_d is the power dissipation, and R_{th} is the thermal resistance in degrees Celsius per watt. The device temperature T_d is

$$\mathrm{T}_d = \Delta T + T_B \tag{2}$$

where T_B is the mounting surface, or baseplate, temperature.

Manuscript received August 3, 2001; revised December 13, 2001.

S. A. Maas is with the Applied Wave Research, Inc., Long Beach, CA 90807 USA.(e-mail: smaas@mwoffice.com).

Publisher Item Identifier S 1531-1309(02)03329-9.

The parameters of the MET model are determined at a temperature $\mathrm{T_{nom}}$, and most parameters depend linearly on $\mathrm{T}_d - \mathrm{T_{nom}}$. In particular, the threshold voltage V_T is

$$V_T = V_{\mathrm{T0}}(1 + \alpha(T_d - \mathrm{T_{nom}})) \tag{3}$$

where V_{T0} is the threshold voltage at $T_d = T_{\mathrm{nom}}$ and α is the temperature coefficient. (Similar linear corrections are used in other models, as well.) Usually, $\mathrm{T_{nom}} \sim T_B$ so for simplicity we can assume that

$$V_T = V_{T0}(1 + \alpha \cdot \Delta T). \tag{4}$$

We now illustrate the problem of ill conditioning by using this threshold-voltage expression in a simple, quadratic FET model. The purpose of this example is to illustrate, in an intuitively comprehensible manner, how ill conditioning can occur; it is not intended to represent a valid thermal model of modern MOSFETs. We consider only dc bias, and we assume that the drain current I_d is given by the expression

$$I_d = \beta\,(V_{\mathrm{gs}} - V_T)^2 = \beta\,(V_{\mathrm{gs}} - V_{T0}(1 + \alpha \cdot \Delta T))^2 \tag{5}$$

where β is a constant and V_{gs} is the gate-to-source voltage. From (1), ΔT is

$$\Delta T = V_{\mathrm{dd}} I_d R_{\mathrm{th}} \tag{6}$$

or, in inverse form,

$$I_d = \frac{\Delta T}{V_{\mathrm{dd}} R_{\mathrm{th}}} \tag{7}$$

where V_{dd} is the drain-to-source voltage.

Equations (5) and (7) can be viewed as functions of ΔT that must be solved simultaneously. A solution can be determined by graphing the two equations and finding the point of intersection of the two curves. Such a plot is shown in Fig. 1. It is clear that, in the case presented, the pair of equations has two solutions, and, because the slopes of the curves are approximately equal, neither is distinct. It represents a classical, ill-conditioned case. Furthermore, if R_{th} were increased slightly, the slope of the I_{d2} curve would decrease, and no solution would exist. This case corresponds to thermal runaway.

On the other hand, if α were negative, the slope of the I_d (5) curve would be negative at the intersection, and a distinct solution would exist. Depending on the other parameter values, a second, spurious solution might exist; however, it would occur at a very high temperature, and the simulator usually would find the correct one.

It is a common practice to limit the maximum value of ΔT in some numerically acceptable manner. This practice prevents transient high temperature values, which might occur during the harmonic-balance iterations, from causing numerical errors in the device equations. This practice has the effect, in Fig. 1, of making the I_d curve flat above the maximum temperature.

1531-1309/02$17.00 © 2002 IEEE

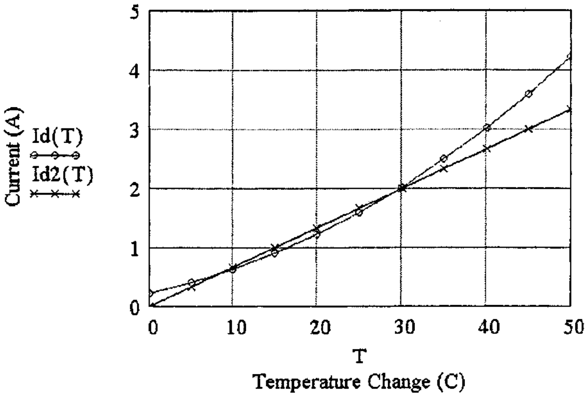

Fig. 1. Plots of I_d from (5) (o) and (7) (x) for the case $\alpha = 0.1$, $R_{\rm th} = 5.0$, $\beta = 0.1$, $V_{T0} = -1.0$, and $V_{\rm dd} = 3.0$. Two solutions exist, neither of which is distinct.

This expedient would create yet a third solution of the system of equations, and, thus, might do more harm than good.

In summary, numerical ill conditioning can occur when the derivatives of (5) and (7) are approximately equal over a broad range ($\sim$ tens of degrees) of ΔT. A sufficient condition for nonexistence of a solution is that the derivative of (7) be less than that of (5) over the entire temperature range.

We have found that, in many devices, the negative shift in threshold voltage with increased temperature is a dominant effect in causing such ill conditioning. Another strong effect is the reduction in mobility with increased temperature, which tends to reduce I_d and increase contact resistances. In the above example, mobility reduction would increase the ill conditioning; however, in more realistic cases, such as the MET example below, it tends to reduce the problem. In most devices, the interplay of these phenomena may either reduce or exacerbate the problem of poor conditioning.

III. MET Numerical Experiment

A numerical experiment illustrates this phenomenon in a practical FET model. Although the MET model was used, any FET model satisfying the conditions stated above can exhibit the same ill conditioning. $I_d(\Delta \mathrm{T})$ is calculated by setting $R_{\rm th} = 0$ in the model and varying the baseplate temperature. Default values (defined in [11]) of all parameters, except for temperature coefficients, are used; only the temperature coefficient of V_T is nonzero.

Fig. 2 shows plots analogous to those of Fig. 1. The limit of convergence was $R_{\rm th} = 25$ C/W; occasionally, convergence was achieved at higher values of $R_{\rm th}$ but in such cases it was clearly because of numerical limiting of the temperature excursions, causing the flat portion of the curve. The model exhibited multiple solutions in this region. It is clear that no practical solution exists (i.e., no solution at a practical temperature) at values of $R_{\rm th} > 20$.

IV. Conclusion

Self heating effects in FET models can cause severe ill conditioning, resulting in solutions that are indistinct, multiple, or nonexistent. The result is poor convergence in harmonic-balance simulations using such models. This effect has been demonstrated in the MET model, but other models that include self heating can exhibit the phenomenon. A necessary condition for nonexistence of a solution is that the derivative of (7) be equal to or less than that of (5) over a broad temperature range.

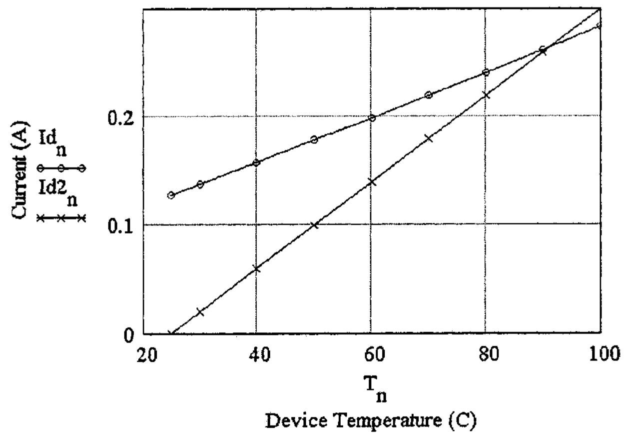

Fig. 2. Plots of $I_d(T)$ (o) and (5) (x) for the MET model. The model parameters are the defaults defined in [11], but all temperature coefficients except $V_{T0} = -0.02$ are set to zero. $R_{\rm th} = 25.0$, $V_{\rm dd} = 10.0$ V, $V_{\rm gs} = 4.0$ V.

Acknowledgment

The author wishes to thank I. Angelov of Chalmers University, and O. Tornblad and C. Blair of Ericsson, for reviewing the manuscript and making helpful comments.

References

[1] J. D. Albrecht *et al.*, “AlGaN/GaN heterostructure field-effect transistor model including thermal effects,” *IEEE Trans. Electron Devices*, vol. 47, p. 2031, 2000.
[2] N. Rinaldi, “Small-signal operation of semiconductor devices including self-heating, with application to thermal characterization and instability analysis,” *IEEE Trans. Electron Devices*, vol. 48, p. 323, Feb. 2001.
[3] L. Wang *et al.*, “Self-heating and kink effects in a a-Si:H thin film transistors,” *IEEE Trans. Electron Devices*, vol. 47, p. 387, Feb. 2001.
[4] W. Liu, “The use of base ballasting to prevent the collapse of current gain in AlGaAs/GaAs heterojunction bipolar transistors,” *IEEE Trans. Electron Devices*, vol. 43, p. 245, Feb. 1996.
[5] G. Gao *et al.*, “Emitter ballasting design for, and current handling capabilities of AlGaAs/GaAs power heterojunction bipolar transistors,” *IEEE Trans. Electron Devices*, vol. 38, p. 185, Feb. 1991.
[6] V. Rizzoli, A. Costanzo, C. Cecchetti, and A. Chiarini, “A nonlinear model of the power MESFET including temperature and breakdown effects,” in *MTT-S Int. Microwave Symp. Dig.*, 1998, p. 1603.
[7] V. Rizzoli, F. Mastri, A. Neri, and A. Lipparini, “Analysis of electrothermal transients and digital signal processing in electrically and thermally nonlinear microwave circuits,” in *MTT-S Int. Microwave Symp. Dig.*, 1997, p. 367.
[8] C. M. Snowden, “Large-signal microwave characterization of AlGaAs/GaAs HBT’s based on a physics-based electrothermal model,” *IEEE Trans. Microwave Theory Tech.*, vol. 45, p. 58, Jan. 1997.
[9] P. C. Grossman, “Large signal modeling of HBT’s including self-heating and transit-time effects,” *IEEE Trans. Microwave Theory Tech.*, vol. 40, p. 449, Mar. 1992.
[10] W. R. Curtice *et al.*, “A new dynamic electro-thermal nonlinear model for silicon RF LDMOS FET’s,” in *IEEE MTT-S Int. Microwave Symp. Dig.*, 1999.
[11] Motorola’s Electro Thermal (MET) LDMOS Model. [Online]. Available: http://e-www.motorola.com/collateral/LDMOSMODELS-RF.html